..IEORY OF FERMION REGGE POLES

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ABSTRACT

A theory for spinor Regge trajectories is done on the basis of the multiperipheral model. A set of linear integral equations for the corresponding high energy amplitudes is obtained, which shows the characteristic $s^{\alpha(t)}$ behaviour. The eigenvalue problem presents new features which are directly connected with the existence of spin. Indeed, there are two dominant trajectories, one being the complex conjugate of the other. In the weak coupling limit, explicit expressions for the trajectories are given : they are real in the bound state region $(t > 0)$ and coincide for $t = 0$. The relation of our results and the properties of bound states is discussed. The complex conjugate character of the trajectories in the diffractive region is found to be related to the existence of two parity bound states for any particular total angular momentum.

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1. INTRODUCTION

Recently Gribov [1] has studied the high-energy behaviour of backward pion-nucleon scattering, assuming that it is dominated by the exchange of a Regge pole [see Fig. 1] which must have fermion characteristics.

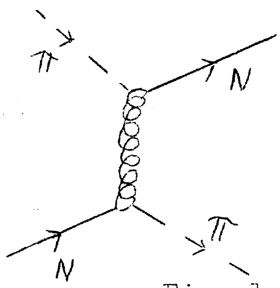

Fig. 1

He derived the properties of the Regge pole trajectories using the analytic properties of the scattering amplitudes suggested by the Mandelstam representation and the hypothesis that the trajectories are real in the bound state region. He showed that in the physical region for backward scattering there will be two complex conjugate Regge trajectories, which coincide when the (crossed) momentum transfer (t) is zero.

The fact that the nucleon spin can give rise to new analytic properties of the Regge trajectories seems surprising at first sight. One can therefore ask, first, whether these properties are obtained in alternative approaches to the high-energy behaviour, and, secondly, to what extent the spin is responsible for the trajectories being complex.

Recently, a relativistic theory [2] was proposed for investigating the asymptotic properties of scattering amplitudes starting from a model for inelastic processes at high energies. This theory predicted the Regge behaviour for two-body processes. The aim of this paper is to use this model to examine the exchange of a spinor Regge pole.

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The theory in question (multiperipheral model) consists in computing the sum of the ladder graphs shown in Fig. 2, by obtaining an integral equation for the sum *).

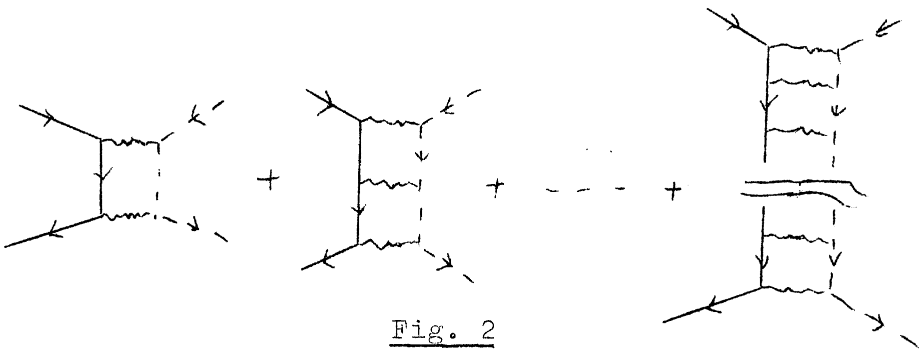

Fig. 2

In Fig. 2 a pion (dotted line) and a nucleon (full line) exchange a series of low-energy systems. In the scalar case (paper I), the asymptotic integral equation for the absorptive amplitude allowed solutions of the form $s^{\alpha(t)}$. The function $\alpha(t)$ was given by an eigenvalue problem. Moreover, the integral equation in question, when continued to the bound state region, coincides exactly (when α was an integer) with the Bethe-Salpeter equation for bound state of angular momentum α.

This fact allowed us to understand, besides the asymptotic properties of scattering, the relation of these with the bound state problem.

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*) We refer to paper I for the justification and the physical meaning of the theory as well as for the notation and details of the method.

In the particular case of the weak coupling limit, an explicit method of solution of the eigenvalue problem was obtained [3], including a definite expression for $\alpha(t)$.

We shall show in the following how all these features are easily extended for the spin case in which we are interested. The eigenvalue problem will present new features which are directly connected with the existence of the spin. Indeed we find two types of solution α^+ and α^- which are related by $\alpha^+(\sqrt{t}) = \alpha^-(-\sqrt{t})$. This relation contains already the Gribov result. In particular, we shall obtain the dominant trajectories explicitly in a weak coupling approximation. There are indeed two complex conjugate trajectories in the physical region for backward πN scattering; and these trajectories become real and inequal in the region of πN bound states.

II. INTEGRAL EQUATION FOR THE ABSORPTIVE AMPLITUDE

Following the discussion given in the introduction, the integral equation for the absorptive $N\bar{N} \rightarrow \pi\pi$ amplitude $M(p_1q_1, p_2q_2)$ can be represented graphically as :

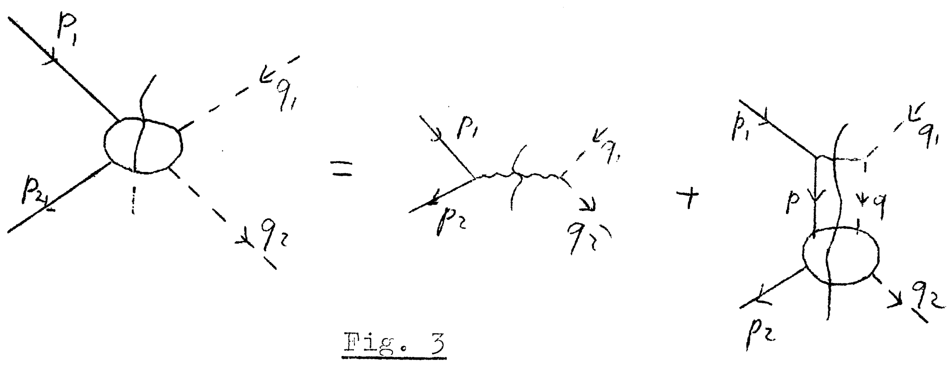

Fig. 3

where p's represent nucleon momenta and q's the pion ones.

In order to solve the integral equation, we shall allow the nucleon (p_1) and the pion (q_1) to be virtual. Defining

$$s = (p_2 - p_1)^2$$
$$t = (p_2 + q_2)^2$$
$$p_1^2 = -v \qquad\qquad q_1^2 = -u$$

the integral equation is

$$M(p_1, q_1; p_2, q_2) = \pi G g\, \delta(s - s_0) + \frac{Gg}{16\pi^3} \int d^4p \frac{m + \not{p}}{(p^2 - m^2)(q^2 - \mu^2)} M(p, q; p_2, q_2)\, \delta[(p - p_1)^2 - s_0] \qquad (1)$$

For simplicity we have considered the Born approximation to be

$$M^R = \pi G g\, \delta(s - s_0)$$

which corresponds to the exchange of a scalar particle with mass $\sqrt{s_0}$ coupled with the nucleon and the pion with coupling constants G and g respectively. In other words, we are considering the ladded approximation for the $N\bar{N} \rightarrow \pi\pi$ amplitude for an interaction Hamiltonian $H' = G\bar{\Psi}\Psi\varphi + g\, \phi_\pi^2 \varphi$. M is a matrix in the spin space of the nucleon. Let us develop it in terms of invariant combinations of γ matrices. In doing so, we note that eventually we shall be interested only in the matrix elements of M between spinors $\bar{u}(p_1)$ and $u(p_2)$ representing the incoming nucleon and antinucleon, respectively; these spinors satisfy $(\not{p}_2 - m)u(p_2) = 0$ and $\bar{u}(p_1)(\not{p}_1 - m) = 0$ when also the nucleon p_1 is on the mass shell. In general, we shall have four independent invariants, we choose to be :

$$\begin{aligned} O_1 &= 1 \\ O_2 &= \not{q}_2 \\ O_3 &= \not{p}_1 - m \\ O_4 &= (\not{p}_1 - m)\not{q}_2 \end{aligned} \qquad (2)$$

so that :

$$M(p_1, q_1; p_2, q_2) = \sum_{i=1}^{4} \mathcal{Q}_i(s, u, v; t)\, O_i(p_1, q_2) \tag{3}$$

On the mass shell we have the identification with the usual definition

$$\begin{aligned} M &= \mathcal{A} + \mathcal{D}\cancel{Q} \\ \mathcal{A} &= \mathrm{Im}\, A = \mathcal{Q}_1 \\ \mathcal{D} &= \mathrm{Im}\, B = \mathcal{Q}_2 \end{aligned} \tag{4}$$

The integral equation can now be written as

$$\sum_1^4 \mathcal{Q}_i(s,u,v;t)\, O_i(p_1,q_2) = -\pi G g\, \delta(s-s_0) + \frac{G g}{16\pi^3} \int \frac{ds'\, du'\, dv'}{(u'+\mu^2)(v'+m^2)} \cdot$$

$$\cdot \sum_{j=1}^{4} \mathcal{Q}_j(s',u',v';t) \int d^4p\; \delta(v'+p^2)\, \delta(u'+q^2)\, \delta(s'-(p-p_2)^2)\, \delta(s_0-(p-p_1)^2)\, (\not{p}+m)\, O_j(p,q_2) \tag{5}$$

In order to obtain equations for the $\mathcal{Q}_i$ we must expand *) $(\not{p}+m)O_j(p,q_2)$ in terms of the invariants $O_i(p_1,q_2)$. We have :

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*) To accomplish this expansion, one must be able to express p in terms of p_1, p_2, q_1 and q_2. However, by momentum conservation, only three of p_1, p_2, q_1, q_2 are independent and to expand p we must introduce a vector r orthogonal to these three. It is easy to see that apart from the factor $(\not{p}+m)\, O_j(p,q_2)$ the integrand in Eq. (5) is symmetric in the component of p in the direction of r; hence terms linear in r integrate to zero. Using this result, and the fact that p_2 is on the mass shell, one obtains the expansion of $(\not{p}+m)\, O_j(p,q_2)$ in terms of the $O_i(p_1,q_2)$ [and verify that no new invariants are necessary].

$$(\not{p}+m)\, O_j(p,q_2) = \sum_{k=1}^{4} C_{kj}(u,v,u',v',s,s_0,s';t)\, O_k(p_1,q_2) \tag{6}$$

In the asymptotic limit introduced in I, namely s large, s' ranging from low values up to order s, while masses u,v,u',v' and t are of the order m, the coefficients of the expansion are

$$C_{kj} = \begin{pmatrix} m(\rho+x+1) & (t-m^2)\rho & -(v'+m^2) & 0 \\ \rho & m(1+x-\rho) & 0 & -(v'+m^2) \\ x & 0 & 0 & 0 \\ 0 & x & 0 & 0 \end{pmatrix} \tag{7}$$

where

$$\rho = \frac{t-v'+u'+x(-t+v-u)}{2t} \tag{8}$$

and $x = \frac{s'}{s}$.

Equating the coefficients of the different invariants, the integral equations in the asymptotic limit are

$$a_i(s,u,v,t) = \frac{Gg}{16\pi^3}\int \frac{dx\,du'\,dv'}{(u'+\mu^2)(v'+m^2)} \sum_{j=1}^{4} C_{ij}(u,v,u',v',x;t)\, a_j(s',u',v',t)\, \times$$
$$\times\, K(u,v,u',v',x;t) \tag{9}$$

where K is the kernel one obtains in the scalar case

$$K(u,v,u',v';t) = \int d\vec{q}\,\delta\left[(\vec{r}+\vec{q})^2 - u' + ux + \frac{s_0 x}{1-x}\right]\delta\left[(\vec{r}-\vec{q})^2 - v' + vx + \frac{s_0 x}{1-x}\right] \tag{10}$$

$$\vec{r}^{\,2} = -\frac{t}{4}(1-x)$$

We note that the integral equations are invariant under the translation $s \rightarrow cs$ and $s' \rightarrow cs'$; therefore a general solution will be

$$a_i(s,u,v,t) = \varphi_i(uvt)\, s^{\alpha(t)} \tag{11}$$

Introducing this solution in the integral equation (9) we obtain the following system of four coupled integral equations for the φ_i

$$\varphi_i(uvt) = \frac{Gg}{16\pi^3}\int_0^1 dx\, x^{\alpha(t)} \iint \frac{du'\,dv'}{(u'+\mu^2)(v'+m^2)} \sum_{j=1}^{4} C_{ij}(u,v,u',v',x,t)\, \times$$

$$\times\, K(u,v,u',v',x;t)\, \varphi_j(u'v't) \tag{12}$$

This system of linear homogeneous Fredholm equations is a typical eigenvalue problem, whose solution will give us both $\alpha(t)$ and φ_i. The solution of the system can be simplified by noting that it can be split into two uncoupled systems of two integral equations for the following combinations of the functions φ_i

$$\begin{aligned}
\phi_1 &= \varphi_1 + (\sqrt{t} - m)\,\varphi_2 \\
\phi_2 &= \varphi_1 - (\sqrt{t} + m)\,\varphi_2 \\
\phi_3 &= \varphi_3 + (\sqrt{t} - m)\,\varphi_4 \\
\phi_4 &= \varphi_3 - (\sqrt{t} + m)\,\varphi_4
\end{aligned} \tag{13}$$

calling

$$\tilde{a}_i(s,u,v,t) = \phi_i(uvt)\, s^{\alpha(t)} \tag{13'}$$

For such amplitudes, the system of integral equation reduces to

$$\begin{cases} \phi_1(uvt) = \dfrac{Gg}{16\pi^3}\displaystyle\int_0^1 dx\, x^{\alpha(t)} \iint \dfrac{du'dv'\, K(u,v,u',v',x;t)}{(u'+\mu^2)\,(v'+m^2)} \Big[(m(1+x)+g\sqrt{t})\,\phi_1(u'v't) - \\ \qquad - (v'+m^2)\,\phi_3(u'v't)\Big] \\ \phi_3(uvt) = \dfrac{Gg}{16\pi^3}\displaystyle\int_0^1 dx\, x^{\alpha(t)+1} \iint \dfrac{du'dv'\, K(u,v,u',v',x;t)}{(u'+\mu^2)\,(v'+m^2)}\, \phi_1(u'v't) \end{cases} \tag{14a}$$

$$\begin{cases} \phi_2(uvt) = \dfrac{Gg}{16\pi^3}\displaystyle\int_0^1 dx\, x^{\alpha(t)} \iint \dfrac{du'dv'\, K(u,v,u',v',x;t)}{(u'+\mu^2)\,(v'+\mu^2)} \Big[(m(1+x)+g\sqrt{t})\,\phi_2(u'v't) - \\ \qquad - (v'+m^2)\,\phi_4(u'v't)\Big] \\ \phi_4(uvt) = \dfrac{Gg}{16\pi^3}\displaystyle\int_0^1 dx\, x^{\alpha(t)+1} \iint \dfrac{du'dv'\, K(u,v,u',v',x;t)}{(u'+\mu^2)\,(v'+m^2)}\, \phi_2(u'v't) \end{cases} \tag{14b}$$

The possibility of splitting the system of integral equations evidently follows from a symmetry of the problem, whose physical meaning we shall discuss later.

Let us refer with a + or - index to the solution of the systems (14a) and (14b) respectively, and let us define

$$\begin{aligned} \tilde{a}^{+}(suvt) &= \begin{pmatrix} \tilde{a}_1 \\ \tilde{a}_3 \end{pmatrix} = \begin{pmatrix} \phi_1(uvt) \\ \phi_3(uvt) \end{pmatrix} s^{\alpha^{+}(t)} \\ \tilde{a}^{-}(suvt) &= \begin{pmatrix} \tilde{a}_2 \\ \tilde{a}_4 \end{pmatrix} = \begin{pmatrix} \phi_2(uvt) \\ \phi_4(uvt) \end{pmatrix} s^{\alpha^{-}(t)} \end{aligned} \tag{15}$$

We note that the system (14a) converts into (14b) for $\sqrt{t} \to -\sqrt{t}$ and vice versa. Therefore the solutions will be related by

$$\tilde{a}^{+}(\sqrt{t}) = \tilde{a}^{-}(-\sqrt{t}) \qquad \text{and} \qquad \alpha^{+}(\sqrt{t}) = \alpha^{-}(-\sqrt{t}) \tag{16}$$

Besides, the kernels of the integral equations are real except for the $\sqrt{t}$ terms. Therefore if $\tilde{a}(\sqrt{t})$ is a solution of the type (+), $\tilde{a}^{*}(\sqrt{t})$ is a solution of the type (-). As a consequence, for t negative, we shall have in general pairs of complex conjugate (equally dominant) trajectories of the form

$$\begin{aligned} \alpha^{+}(\sqrt{t}) &= \alpha_1(t) + \alpha_2(t)\sqrt{t} \\ \alpha^{-}(\sqrt{t}) &= \alpha_1^{*}(t) - \alpha_2^{*}(t)\sqrt{t} \end{aligned} \tag{17}$$

Eq. (17) contains the Gribov result. In fact, real trajectories in the bound state region $(t > 0)$ imply α_1 and α_2 real and the two complex conjugate trajectories coincide for $t=0$. Before discussing the physical meaning of our result, we shall obtain the explicit solution of the systems (14a) and (14b) for $\alpha^{\pm}$ in the weak coupling limit $Gg \to 0$.

III. THE WEAK COUPLING

As discussed in Ref. [3], the weak coupling limit can be obtained by considering the kernel of Eqs. (14) at its value for $x=0$. To begin with, we want to stress the fact, already pointed out in Ref. [3], that the weak coupling limit of our equations for the asymptotic absorptive amplitude just coincides with the sum of the asymptotic perturbative expressions for the ladder diagrams (or rather for the imaginary part of them) of

Fig. 2. In the general strong coupling case treated before, due to the higher multiperipheral graphs of Fig. 2, it is no longer true that the asymptotic behaviour of the sum is equal to the sum of the asymptotic behaviours.

In the weak coupling limit the system (14) reduces to :

$$\phi_3 = \phi_4 = 0$$

$$\phi_1(uvt) = \frac{Gg}{16\pi^3}\frac{1}{\alpha^+ + 1}\iint \frac{du'dv'\,K(u,v,u',v',o;t)}{(u'+\mu^2)(v'+m^2)}(m+\rho\sqrt{t})\,\phi_1(u'v't) \tag{18}$$

$$\phi_2(uvt) = \frac{Gg}{16\pi^3}\frac{1}{\alpha^- + 1}\iint \frac{du'dv'\,K(u,v,u',v',o;t)}{(u'+\mu^2)(v'+m^2)}(m-\rho\sqrt{t})\,\phi_2(u'v't)$$

From the explicit expressions for ρ and K given in (8) and (10), it appears that both K(u,v,u',v',o;t) and ρ(u,v,u',v',o;t) do not depend on u and v, so that also the solutions of (18) shall not depend on such variables, i.e.,

$$\phi_1(uvt) = C^+(t)$$
$$\phi_2(uvt) = C^-(t) \tag{19}$$

Defining

$$F_1(t) = \frac{1}{2\pi}\iint \frac{du'dv'\,K(u,v,u',v',o;t)}{(u'+\mu^2)(v'+m^2)} = \frac{2\log\left(\dfrac{\sqrt{|t|+(m-\mu)^2}+\sqrt{|t|+(m+\mu)^2}}{2\sqrt{m\mu}}\right)}{\left\{\left[|t|+(m-\mu)^2\right]\left[|t|+(m+\mu)^2\right]\right\}^{1/2}}$$

$$F_2(t) = \frac{1}{2\pi}\iint \frac{du'dv'\,K(u,v,u',v',o;t)}{(u'+\mu^2)(v'+m^2)}\left(\frac{u'-v'}{t}\right) = -\frac{1}{|t|}\left(\log\frac{\mu}{m} + (m^2-\mu^2)F_1(t)\right) \tag{20}$$

due to the homogeneity of Eqs. (18), we obtain

$$\alpha^{\pm}(t) = -1 + \frac{Gg}{16\pi^3}\left[m\,F_1(t) \pm \frac{\sqrt{t}}{2}\left(F_1(t)+F_2(t)\right)\right] \tag{21}$$

The coefficients $C^+(t)$ and $C^-(t)$ can be obtained by normalizing to the asymptotic expression of the first peripheral term (4th order). This term can easily be obtained by replacing α_j by its Born approximation $-\delta_{i1}\ Gg\,\delta(s'-s_0)$ in the r.h.s. of Eq. (9) and by performing then the linear combinations analogous to (13). By comparing then with Eq. (15), taking into account (19) and (21), we would obtain

$$C^{\pm}(t) = -\frac{(Gg)^2}{16\pi}\left[m\,F_1(t) \pm \frac{\sqrt{t}}{2}\left(F_1(t)+F_2(t)\right)\right] \tag{22}$$

Comparing (21) with (22), it appears that here, as in the scalar case, the Regge trajectory for $\alpha^{\pm}(t)+1$ and the energy independent coefficients $C^{\pm}(t)$ are proportional to each other in the weak coupling limit. It is interesting to note that in such a limit the asymptotic behaviour of the amplitude is completely independent of the mass $\sqrt{s_0}$ of the exchanged particle (i.e., the range of the potential).

By comparing (20) with the general form (17), we see that in the weak coupling limit, the coefficients $\alpha_1(t)$ and $\alpha_2(t)$ are indeed real, so that the trajectories are complex conjugate of each other for $t<0$, coincide at $t=0$ and are real and unequal for $t>0$. The reality of the coefficients imply that the equal dominant complex conjugate trajectories are related among them by $\alpha^+(\sqrt{t}) = \alpha^-(-\sqrt{t})$.

A recent letter by Gell-Mann and Goldberger [4] discussed (but did not test) the possibility that the nucleon itself lies on a Regge trajectory, in perturbation theory. This requires that there be radiative corrections to the backward pion-nucleon scattering amplitude which behave as s^0 when s is large. We have discussed only radiative corrections behaving as s^{-1}; in effect we assume that the nucleon pole is a fixed pole with $\alpha=0$, and the trajectory we have computed is the one with $\alpha \to -1$ as $Gg \to 0$.

IV. DISCUSSION OF THE RESULTS

Eq. (17) shows that the spinor character of the problem leads necessarily to two types of trajectories, α^+ and α^-, complex conjugate of each other and showing a $\sqrt{t}$ singularity. In order to discuss the physical meaning of this result, let us obtain the physical on the mass shell amplitudes, restricting to the dominant asymptotic solutions of (14). On the mass shell the amplitudes $\mathcal{a}_3$ and $\mathcal{a}_4$ of Eq. (3) do not contribute; the absorptive parts of the usual amplitudes A and B being given by (4). From (11), (13) and (13'), we have that, asymptotically,

$$\mathcal{A} = \frac{\sqrt{t}+m}{2\sqrt{t}}\,\tilde{\mathcal{a}}_1(s,-\mu^2,-m^2;t) + \frac{\sqrt{t}-m}{2\sqrt{t}}\,\tilde{\mathcal{a}}_2(s,-\mu^2,-m^2;t)$$

$$\mathcal{B} = \frac{\tilde{\mathcal{a}}_1(s,-\mu^2,-m^2;t) - \tilde{\mathcal{a}}_2(s,-\mu^2,-m^2;t)}{2\sqrt{t}} \qquad (23)$$

where we remember that $\tilde{\mathcal{a}}_1$ and $\tilde{\mathcal{a}}_2$ - defined in (13') - are given by the solutions of the systems (14a) and (14b) respectively.

As we discussed in Section II, in the diffractive $t < 0$ region,

$$\tilde{\mathcal{a}}_1(st) = \tilde{\mathcal{a}}_2^*(st)$$

In order to calculate the real parts of A and B, we can write for them fixed t dispersion relations. Or, equally well, we can write the dispersion relations for the amplitudes

$$\mathcal{F}_1(st) = A(st) + (\sqrt{t}-m)\,B(st)$$

$$\mathcal{F}_2(st) = A(st) - (\sqrt{t}+m)\,B(st) \qquad (24)$$

whose discontinuities are just $\tilde{\mathcal{a}}_1$ and $\tilde{\mathcal{a}}_2$ respectively.

The absorptive parts we have calculated before are those for s positive and big, being clear that the multiperipheral graphs of Fig. 2 do not contribute to the negative s cut. We shall discuss later the possible multiperipheral model for such a contribution and its implication. Let us consider for the moment the simple case in which we consider only the positive s cut as given by (13'). We can then easily obtain the asymptotic behaviour for ReA and ReB for small negative t and large positive (asymptotic $N\bar{N} \rightarrow \pi\pi$) or negative s (backward π N scattering) to be the following

$$\mathrm{Re}\,A = -\frac{\sqrt{t}+m}{2\sqrt{t}}\,\phi_1(\sqrt{t})\, s^{\alpha^{+}(\sqrt{t})} \cot \pi\alpha^{+}(\sqrt{t}) + \text{c.c.}$$
$$\mathrm{Re}\,B = -\frac{\phi_1(\sqrt{t})\, s^{\alpha^{+}(\sqrt{t})}}{2\sqrt{t}} \cot \pi\alpha^{+}(\sqrt{t}) + \text{c.c.} \qquad (25)$$

Let us now understand the physical reason due to which the linear combination of amplitudes of Eq. (13) splits into two systems of integral equations. We realize now that the amplitudes $\mathcal{F}_1$ and $\mathcal{F}_2$ defined in (24) are just proportional to the usually defined non-covariant amplitudes f_1 and f_2 [5)] which, in the t channel, have the following partial wave decomposition [*)]

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*) E and θ_t are the c.m. nucleon energy and scattering angle in the t channel, respectively, given by :

$$E = \frac{t+m^2-\mu^2}{\sqrt{t}} \quad , \quad \cos\theta_t = 1 + \frac{st}{\frac{1}{2}(t-m^2-\mu^2)^2 - 2\mu^2 m^2}$$

$$f_1 = \frac{E+m}{8\pi\sqrt{t}}\,\mathcal{F}_1(s\,t) = \sum_J f_{J^-}(t)\,P'_{J+1/2}(\cos\theta_t) - f_{J^+}(t)\,P'_{J-1/2}(\cos\theta_t)$$

$$f_2 = \frac{-E+m}{8\pi\sqrt{t}}\,\mathcal{F}_2(s\,t) = \sum_J f_{J^+}(t)\,P'_{J+1/2}(\cos\theta_t) - f_{J^-}(t)\,P'_{J-1/2}(\cos\theta_t) \qquad (26)$$

where

$$f_{J^\pm} = e^{i\delta_{J^\pm}}\sin\delta_{J^\pm} \qquad (27)$$

$\delta_{J\pm}$ being the usual π N phase shifts for total angular momentum J and $\ell = J \pm 1/2$.

If we would perform over the expressions of Eq. (26) the usual Regge procedure of converting the sum in an integral and then consider the asymptotic behaviour $\cos\theta_t \to \infty$ we could see that asymptotically, and up to relative order $1/s$,

$$\begin{cases} f_1 \propto s^{\alpha} \\ f_2 \sim 0 \end{cases} \qquad (28a)$$

imply a family of bound states with $\ell = J - 1/2$ for $J = \alpha + \frac{1}{2}$.

$$\begin{cases} f_1 \sim 0 \\ f_2 \propto s^{\alpha} \end{cases} \qquad (28b)$$

imply a family of bound states with $\ell = J + 1/2$ for $J = \alpha + \frac{1}{2}$.

From the proportionality of f_1 and f_2 with $\mathcal{F}_1$ and $\mathcal{F}_2$, we can immediately relate our solution α^+ as the continuation to negative t of the trajectory of the family of bound states with $J = \ell + \frac{1}{2}$, and our α^- to those with $J = \ell - \frac{1}{2}$.

We understand therefore that the amplitudes which decouple our system of integral equations and have complex conjugate asymptotic behaviour for $t < 0$, are just those amplitudes that, when considered on the mass shell and decomposed into partial waves in the t channel $(t > 0)$, have for a given angular momentum J a well defined parity (i.e., well defined angular momentum) for $\cos \theta_t \to \infty$.

We understand therefore the following fact. The translational symmetry which allows us to select the asymptotic behaviour $s^{\alpha(t)}$ is the correspondent, in the diffraction region, of the rotation symmetry in the bound state region. Therefore, here - as in the scalar case - its eigenvalue α corresponds to the total angular momentum J for bound states. When J is selected, the subsequent choice of the parity (choice of ℓ) corresponds in the diffraction region to the diagonalization of system (14) for a particular α.

Summarizing the preceding discussion, we have shown that there exist two different Regge pole trajectories for a fermion problem. In the diffractive region they are complex conjugate of each other and give the asymptotic behaviour of the fermionic exchange scattering amplitudes. For $t > 0$ their intersection with an integer value of α represents a bound state with $J=\alpha+1/2$ and $\ell=\alpha$ if the trajectory is the $(+)$ and $\ell=\alpha+1$ if it is the $(-)$ one.

Our trajectories do not possess a signature character or, if we prefer, the trajectories corresponding to the opposite signatures are degenerate. This is due to the fact, already pointed out before, that we have not considered the imaginary amplitudes for negative s (backward πN scattering) or, in a different language, that we have considered no exchange potential. In fact, the imaginary part for backward πN scattering would appear naturally in our model from multiperipheral graphs of the type

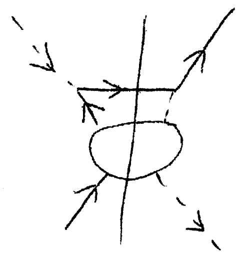

Fig. 4

in which the nucleon itself acts as a potential and which we have omitted for the sake of simplicity.

It can easily be seen that the inclusion of such an imaginary part for $s < 0$ would split the trajectories of different signature. Indeed one can obtain two independent systems of integral equations in which the kernels (Born approximations) are the sum and the difference, respectively, of kernels corresponding to the two multiperipheral diagrams.

Our interest in the problem came mainly from discussions aroused in the Theoretical Study Division on high-energy physics at CERN; we thank therefore the participants and, particularly, Drs. Fubini, Kinoshita and Martin. We are also indebted to Drs. V.N. Gribov and I.Ia. Pomeranchuk for an interesting discussion on the argument to which this paper is referred.

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